

Chemical Engineering Journal 88 (2002) 21–26

Chemical Engineering Journal

www.elsevier.com/locate/cej

# Mixing pool model for prediction of distillation tray efficiency with liquid entrainment

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Received 26 February 2001; received in revised form 13 August 2001; accepted 12 September 2001

## **Abstract**

In order to make clear the effect of liquid entrainment on the distillation tray efficiency, a new type of mixing pool model is developed in this paper to calculate the apparent distillation tray efficiencies when the liquid entrainment happens for the counter current tray with vapour unmixing between trays and liquid partially mixing on trays. The analytical solutions are obtained and compared with the calculated results of Bennett's experimental correlations. The mean relative error between them is 5.6% for the range of  $0 \le Pe \le 1000$ ,  $0.5 \le \lambda \le 3.0$  and  $0.4 \leq E \leq 1.0$ . The result shows that the model presented in this paper is reliable.

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*Keywords:* Distillation; Tray; Mixing pool model; Liquid entrainment; Apparent tray efficiency; Analytical solution

## **1. Introduction**

In industrial columns there are three flow regimes that are encountered commonly—spray, froth and emulsion [1,2]. For normal systems it is often in spray regime when the operation is at low pressures. In this regime liquid on trays is often entrained from the active area to the trays above. When the liquid entrainment is serious, jet flooding will be caused and the operation will be spoiled.

Over the years a great deal of research has been made on the effect of liquid entrainment on the distillation tray efficiencies. Many correlations have been proposed, but most of them fit only for the ideal conditions of liquid completely mixing or being in plug flow on the tray. The first successful attempt to calculate the effect of liquid entrainment on the tray efficiency was made by Colburn [3]. His well-known equation is simple and convenient but valid only when the liquid on the tray mixes completely and  $\lambda_0 = 1$ . Kageyama [4] attempted to obtain a correlation based on the assumption that the vapour is completely mixed between trays but the entrained liquid is unmixed. Lockett [5] has also studied the ideal condition that liquid is in plug flow on trays with vapour unmixing between trays and the analytical solutions were given.

But the more realistic case is that the liquid partially mixes on the tray. In general, there are two kinds of methods to deal with it—mixing pool model and eddy dispersion model. Lockett [6] established eddy dispersion models to calculate the effect of vapour entrainment on the tray efficiency for three cases: case I: liquid partially mixed in counter flow on successive plates with vapour completely mixing between trays; case II: liquid partially mixed in the same direction on successive plates with vapour unmixing between trays; case III: Liquid partially mixed in counter flow on successive plates with vapour unmixing between trays. And the analytical solution was obtained for case II and the results of numerical solutions were given for case I and III by Lockett et al.

Of the three cases, case III is mostly encountered in real operation. However, it is inconvenient to use the numerical solutions in real column design. In this paper a new model is established using a series of mixing pools to stimulate the real liquid flow on the tray and an attempt is made to derive the analytical solutions for case III to predicate the effect of entrainment on distillation tray efficiencies.

In the development of the mixing pool models, the following assumptions are used:

- 1. The flow rate of vapour, liquid and entrainment are constant from tray to tray.
- 2. The vapour–liquid equilibrium relationship can be expressed by:  $y^* = kx + b$ .
- 3. The point efficiency based on the vapour phase is constant.
- 4. The same tray and point efficiency is considered for all components in the mixture.

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## **Nomenclature**





### **2. Division of mixing pools**

In the development of the efficiency models the tray is assumed to be divided into mixing zones. The liquid enters mixing zones in turn. The vapour and entrained liquid entering each zone is equal in amount. In each zone the liquid is completely mixed. The vapour entering and leaving the zones carries the same amount of liquid. The entrained liquid is assumed to be mixed well with the liquid entering the zone and they have the same composition. In flow

direction on tray *n* mixing pools are given serial number to separate from  $\langle n, 1 \rangle$  to  $\langle n, m \rangle$ , where *n* is the serial number of the tray. This model is schematically illustrated in Fig. 1.

## **3. Definition of apparent efficiency**

The definition of apparent efficiency is also an important aspect in the development of the mixing pool model. Consider a column operating without and with entrainment. In each case the vapour flow rate is *V* but entrainment causes internal liquid circulation within the column so that the liquid flow rate  $L_0$  on the tray is increased to  $L_0 + E^L$ .

Then a material balance for the more volatile component around tray *n* can be given as:

$$
V(\bar{y}_n - \bar{y}_{n-1}) + E^{L}(\bar{x}_n - \bar{x}_{n-1}) - (L_0 + E^{L})
$$
  
 
$$
\times (x_{n+1,m} - x_{n,m}) = 0
$$

$$
V(\bar{y}_n - \bar{y}_{n-1}) = L_0 \left\{ \left[ x_{n+1,m} + \frac{E^L}{L_0} (x_{n+1,m} - \bar{x}_n) \right] - \left[ x_{n,m} + \frac{E^L}{L_0} (x_{n,m} - \bar{x}_{n-1}) \right] \right\}
$$

Define an apparent vapour composition  $\bar{y}_n^a$  and an apparent liquid composition  $\bar{x}_n^a$  of tray *n* as follows:

$$
\bar{y}_n^a = \bar{y}_n \tag{1}
$$

$$
\bar{x}_{n+1}^{\text{a}} = x_{n+1,m} + \frac{E^{\text{L}}}{L_0} (x_{n+1,m} - \bar{x}_n)
$$
\n(2)

Then the material balance can be rewritten as:

$$
V(\bar{y}_n^a - \bar{y}_{n-1}^a) = L_0(\bar{x}_{n+1}^a - \bar{x}_n^a)
$$
 (3)

The apparent efficiency  $E_{MV}^{a}$  is then defined as:

$$
E_{\text{MV}}^a = \frac{\bar{y}_n^a - \bar{y}_{n-1}^a}{(\bar{y}_n^*)^a - \bar{y}_{n-1}^a}
$$
(4)

where

$$
(\bar{y}_n^*)^a = k\bar{x}_n^a + b \tag{5}
$$

The other two efficiencies used in models are Murphree tray efficiency and Murphree point efficiency. They are defined as:

$$
E_{\rm M} = \frac{\bar{y}_n - \bar{y}_{n-1}}{y_{n,m}^* - \bar{y}_{n-1}}
$$
(6)

$$
E = \frac{y_{n,1} - y_{n-1,m+1-1}}{y_{n,1}^* - y_{n-1,m+1-1}}
$$
(7)

### **4. Mixing pool model with entrainment for case III**

A new mixing pool model is established to calculate the effect of entrainment on tray efficiency for a tower operating



Fig. 1. Mixing pool model with liquid entrainment: (a) schematic diagram of mixing pools on countercurrent trays; (b) schematic diagram of material balance of the mixing pool.

at total reflux where the liquid on the tray partially mixes and the vapour unmixes between trays. A set of equations are conduced for the top tray, the general tray, the bottom tray, the top and the bottom of tower.

# *4.1. Top tray*

The material balance for any a mixing pool on top tray is:

$$
(L_0 + E^{L})(x_{N,1-1} - x_{N,1}) + \frac{E^{L}}{m}(\bar{x}_{N-1} - x_{N,1})
$$
  
+ 
$$
\frac{V}{m}(y_{N-1,m+1-1} - y_{N,1}) = 0
$$

because

$$
E = \frac{y_{N,1} - y_{N-1,m+1-1}}{y_{N,1}^* - y_{N-1,m+1-1}}
$$

and

 $y_{N,1}^* = kx_{N,1} + b$ 

then

$$
x_{N,1-1} - x_{N,1} = \frac{1}{Ek} [(y_{N,1-1} - y_{N,1}) - (1 - E) \times (y_{n-1,m+1-1} - y_{n-1,m+2-1})]
$$
(8)

$$
\bar{x}_{N-1} - x_{N,1} = \frac{1}{Ek} [(\bar{y}_{N-1} - y_{N,1}) - (1 - E) \times (\bar{y}_{N-2} - y_{N-1,m+1-1})]
$$
\n(9)

So the equation can be rewritten as:

$$
(L + E^{L})x_{N,0} - \left[\frac{L + E^{L}}{Ek} + \frac{E^{L}}{mEk} + \frac{V}{m}\right]y_{N,1} + \left[\frac{L + E^{L}}{Ek}(1 - E) + \frac{E^{L}}{mEk}(2 - E) + \frac{V}{m}\right]y_{N-1,m} - \frac{E^{L}}{mEk}(1 - E)y_{N-2'1} = 0 \qquad 1 = 1 \tag{10}
$$

$$
\left(\frac{L+E^L}{Ek} + \frac{1-1}{m}\right) y_{N,1-1} - \left[\frac{L+E^L}{Ek} + \frac{E^L}{mEk} + \frac{V}{m}\right] y_{N,1}
$$

$$
- \frac{L+E^L}{Ek} (1-E) y_{N-1,m+2-1}
$$

$$
+ \left[\frac{L+E^L}{Ek} (1-E) + \frac{E^L}{mEk} (2-E) + \frac{V}{m}\right] y_{N-1,m+1-1}
$$

$$
- \frac{E^L}{mEk} (1-E) y_{N-2'1} = 0 \qquad 1 = 2, 3...m \tag{11}
$$

## *4.2. General tray*

Similarly make material balance for mixing pools on general trays. The following equation are obtained:

$$
\frac{L+E^L}{Ek}y_{N,0} - \left[\frac{L+E^L}{Ek}(2-E) + \frac{E^L}{mEk} + \frac{V}{m}\right]y_{N,1} + \left[\frac{L+E^L}{Ek}(1-E) + \frac{E^L}{mEk}(2-E) + \frac{V}{m}\right]y_{N-1,m} - \frac{E^L}{mEk}(1-E)y_{N-2,1} = 0 \qquad 1 = 1 \tag{12}
$$

$$
\frac{L+E^L}{Ek} y_{N,1-1} - \left(\frac{L+E^L}{Ek} + \frac{E^L}{mEk} + \frac{V}{m}\right) y_{N,1}
$$
  
+ 
$$
\left[\frac{E^L}{mEk} (2 - E) + \frac{V}{m} + \frac{L+E^L}{Ek} (1 - E)\right] y_{N-1,m+1-1}
$$
  
- 
$$
\frac{E^L}{mEk} (1 - E) y_{N-2,1} - \frac{L+E^L}{Ek} (1 - E) y_{N-1,m+2-1} = 0
$$
  
1 = 2, 3...m (13)

# *4.3. Bottom tray*

The vapour rising from reboiler to the bottom tray carries no liquid. However, when the vapour leaves the mixing pools on the bottom tray,  $E^L$  mol liquid are entrained by the vapour





and the amount of entrained liquid keeps constant within the tower thenceforward.

According to that the model for the bottom tray are obtained:

$$
\frac{L + E^{L}}{Ek} y_{1,0} - \left[ \frac{L + E^{L}}{Ek} (2 - E) + \frac{V}{m} \right] y_{1,1} + \left[ \frac{L + E^{L}}{Ek} (1 - E) + \frac{V}{m} \right] \bar{y}_{0} = 0 \qquad 1 = 1 \tag{14}
$$

$$
\left(\frac{L + E^{L}}{Ek} - \frac{1 - 1}{mEk} E^{L}\right) y_{1,1-1} - \left(\frac{L + E^{L}}{Ek} + \frac{V}{m} - \frac{1 - 1}{mEk} E^{L}\right) y_{1,1} + \frac{V}{m} \bar{y}_{0} = 0
$$
  
1 = 2, 3 ... m (15)

### *4.4. Top and bottom of the tower*

Make material balances at the top and bottom of the tower, rearrange them and the following equations are obtained:

$$
\left(V + \frac{E^{\rm L}}{Ek}\right)\bar{y}_N - \frac{E^{\rm L}}{Ek}(1 - E)\bar{y}_{N-1} - (V + E^{\rm L})x_{N,0} = 0
$$
\n(16)

$$
\frac{L_0}{Ek}y_{1,m} - \left(L_0 + \frac{1 - E}{Ek}\right)\bar{y}_0 = 0\tag{17}
$$

If the value of  $E^L/L_0, E, \lambda_0$  are known, the total number of the unknown variables is  $Nm + 2$  including  $\bar{y}_0x_{N,0}y_{N,1} \ldots y_{N,m}y_{N-1,1} \ldots y_{N-1,m} \ldots y_{1,1} \ldots y_{1,m}$ . The total number of the equations is also  $Nm + 2$ . Since the total number of the unknown is equal to that of equations, there must be one and only one set of solutions for the equation group. Solve the group of equations and the analytical solutions are obtained.

# **5. Comparison of the solutions with the results of Bennett's correlation**

In order to examine the validity of this model presented in this paper, the analytical solutions of the model are compared with the results of Bennett's correlations [7]. Based on the 156 experiment data sets collected, which covered a wide range of geometry and operating conditions, Bennett commended the following correlations to calculate point efficiencies for cross-flow case with vapour unmixed between trays.

The tray efficiency with liquid entrainment can be calculated by

$$
\frac{E_{\rm MV}(e_0)}{E_{\rm MV}(e_0=0)} = 1 - 0.8E\lambda^{0.543}E^{\rm L}\frac{V}{L_0 + E^{\rm L}}\tag{18}
$$

The tray efficiency without liquid entrainment can be calculated by

$$
E_{\rm MV}(e_0 = 0) = \frac{[1 + (\lambda E/n)]^n - 1}{\lambda} (1 - 0.0335 \lambda^{1.07272} \times E^{2.51844} P e^{0.17524})
$$
(19)

where

$$
Pe = 2(n-1) \tag{20}
$$

With  $E_{MV}$  known, instituting Eq. (19) into Eq. (18), then the point efficiency can be calculated. According to Bennett, the average error between the calculated values and experimental data of point efficiencies is 6.3%, with 83% of the data within 10%.

The solutions of this model  $E_{\text{M1}}^{\text{a}}$  and the calculated results of Bennett's model  $E_{\text{M2}}^a$  are compared in Table 1 under the conditions of vapour unmixing between trays and liquid partially mixing on the tray. It is seen from Table 1 that the analytical solutions of this model are in good agreement with the calculated results of Bennett's model and the mean relative error between them is 5.6% for the range of  $0 <$ *Pe* < 1000,  $0.5 < \lambda < 3.0$  and  $0.4 < E < 1.0$ . That shows that the model presented in this paper is reliable.

According to the results, apparent tray efficiency decreases with liquid entrainment and the decreasing extent is affected by stripping factor and point efficiency. Commonly liquid entrainment has more influences on apparent tray efficiency at high point efficiencies than at low ones. That is to say, the higher the point efficiency is, the faster the apparent tray efficiency decreases. It is also the same with tripping factor.

Another fact reflected in the table is that liquid mixing on the tray is unfavourable for the apparent tray efficiency, since it reduces the average driving force for mass transfer between liquid and vapor phase. Therefore, trays in distillation columns should be designed to try to avoid liquid backmixing on the tray on a large scale.

### **6. Conclusions**

- 1. A new model based on the mixing pool model is established for the tower operating at total reflux where the liquid on the tray partially mixes and vapour unmixes between trays to calculate the effect of liquid entrainment on the tray efficiency.
- 2. The analytical solutions of this model are compared with the calculated values of Bennett's correlations. The mean relative error between them is 5.6% for the range of  $0 <$ *Pe*  $\leq$  1000, 0.5  $\leq \lambda \leq$  3.0 and 0.4  $\leq E \leq$  1.0. The result shows that this model is in good agreement with Bennett's correlations, which proves the model presented in this paper is reliable.
- 3. According to the simulation results, the distillation tray efficiency decreases with the liquid entrainment especially at large  $E$  and  $\lambda_0$ .

## **References**

- [1] M.L. Lockett, Distillation tray fundamentals. P7 P12 P172–4. Cambridge University Press, Cambridge, 1986.
- [2] H.Z. Kister, Distillation Design. McGraw-Hill, New York, 1992, p.332.
- [3] A.P. Colburn, Effect of entrainment on plate efficiency in distillation, Inst. Chem. Eng. Symp. Ser. 28 (1936) 526–530.
- [4] O. Kageyama, Plate efficiency in distillation towers with weeping and entrainment, Inst. Chem. Eng. Symp. Ser. 32.2 (1969) 72–82.
- [5] M.A. Rahman, M.L. Lockett, Efficiency reduction of large distillation tray by entrainment, Inst. Chem. Eng. Symp. Ser. 61 (1981) 111– 125.
- [6] M.L. Lockett, M.A. Rahman, H.A. Dhulesia, The effect of entrainment on distillation tray efficiency, Chem. Eng. Sci. 38 (1983) 661– 772.
- [7] D.L. Bennett, D.N. Watson, M.A. Wiescinski, New correlation for sieve-tray point efficiency, entrainment and section efficiency, AIChE J. 43 (6) (1997) 1611–1626.